Online Appendix to *Fiscal Policy in a Networked Economy* by Flynn, Patterson, and Sturm

A. Omitted Proofs

A.1. Proof of Proposition 1

Proof. This proof is a Corollary of Proposition 8, presented in Online Appendix B.2. From Proposition 8, recall that:

$$dY = \left(I - D\left(I - \widehat{X}\right)^{-1}\right)^{-1} \partial Q, \text{ where } D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^1} & \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^2} \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^1} & \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^2} \end{bmatrix} \tag{OA1}$$

Under Assumption 1, this reduces to:

$$D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 & 0 \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 & 0 \end{bmatrix} \tag{OA2}$$

Simple matrix manipulations show that one may extract just the first $\mathcal{I}^1$ rows:

$$dY^1 = \left(I - C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}\right)^{-1} \partial Q^1 \tag{OA3}$$

□

A.2. Proof of Lemma 1

Proof. Starting from Proposition 1 and using that the modulus of $C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}$ is less than 1, we can express:

$$\begin{aligned} dY^1 &= \sum_{k=0}^{\infty} \left[C^1_{y^1} l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1}\right]^k \partial Q^1 \\ &= \partial Q + \overline{C}^1_{y^1} \hat{m} \sum_{k=0}^{\infty} \left[l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \overline{C}^1_{y^1} \hat{m}\right]^k l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1\right)^{-1} \partial Q^1 \\ \vec{1}' dY^1 &= \vec{1}^T \partial Q + m^T \left(\sum_{k=0}^{\infty} (\mathcal{G}\hat{m})^k\right) \partial y^1 \end{aligned} \tag{OA4}$$

where the last line uses the definitions of $\mathcal{G}$ and ∂y^1, and the fact that $\vec{1}^T \overline{C}^1_{y^1} = \vec{1}^T$ (by construction).

Finally, $\vec{1}^T \partial Q^1 = \vec{1}^T \partial y^1$ because $\vec{1}^T \cdot l^1_{L_1} \widehat{L}_1 (I - \hat{X}^1)^{-1} = \vec{1}^T$, since firms earn zero profits.

□

A.3. Proof of Proposition 2

Proof. Let $b \equiv \vec{1}^T(I - \mathcal{G}\hat{m})^{-1}$ be the vector of Bonacich centralities of households in the income-to-spending network; these are well-defined as we have assumed the modulus of $\mathcal{G}\hat{m}$ is less than one. Let $(b^{next})^T = b^T\mathcal{G}$ be the row vector with n^{th} entry equal to the average Bonacich centrality of the household to whom n's marginal spending flows.

We begin by providing a lemma that exactly decomposes the general equilibrium change in output in terms of Bonacich centralities.

Lemma 2. *For any $x \in \mathbb{R}$, the total change in first-period output due to a partial equilibrium demand shock with unit-magnitude labor income incidence ∂y^1 is equal to*

$$\vec{1}^T dY^1 = (1 + x \cdot \mathbb{E}_{\partial y^1}[m_n]) + \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{next}] - x\right) + \mathbb{C}ov_{\partial y^1}[m_n, b_n^{next}] \tag{OA5}$$

Setting x equal to the $\frac{1}{1-MPC}$ multiplier with the MPC weighted by income y^*, we obtain an exact decomposition in the spirit of Proposition 2.

$$\begin{aligned}\vec{1}^T dY^1 = \; & \underbrace{\frac{1}{1 - \mathbb{E}_{y^*}[m_n]}}_{\textit{Keynesian multiplier}} + \underbrace{\frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y^*}[m_n]}{1 - \mathbb{E}_{y^*}[m_n]}}_{\textit{Incidence effect}} \\ & + \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{next}] - \frac{1}{1 - \mathbb{E}_{y^*}[m_n]}\right)}_{\textit{Biased MPC direction effect}} + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, b_n^{next}]}_{\textit{Homophily effect}}\end{aligned} \tag{OA6}$$

Proof. Note that Proposition 1 implies that the change in output resulting from some shock with unit incidence is given by

$$\vec{1}^T dY^1 = b^T \partial y^1 = \vec{1}^T \partial y^1 + b^T \mathcal{G}\hat{m}\partial y^1 \tag{OA7}$$

Let $b^{nextT} = b^T\mathcal{G}$ be the row vector with i^{th} entry equal to the average Bonacich centrality of the household to which i's marginal spending flows. We then have, for any $x \in \mathbb{R}$:

$$\begin{aligned}\vec{1}^T dY^1 &= 1 + \mathbb{E}_{\partial y^1}[m_n b_n^{\text{next}}] = 1 + \mathbb{E}_{\partial y^1}[m_n] \cdot \mathbb{E}_{\partial y^1}[b_n^{\text{next}}] + \mathbb{C}\text{ov}_{\partial y^1}[m_n, b_n^{\text{next}}] \\ &= (1 + x \cdot \mathbb{E}_{\partial y^1}[m_n]) + \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[b_n^{\text{next}}] - x\right) + \mathbb{C}\text{ov}_{\partial y^1}[m_n, b_n^{\text{next}}]\end{aligned} \tag{OA8}$$

□

We can now prove Proposition 2. First, note that:

$$b_n = 1 + m_n + O(|m|^2) = 1 + \frac{m_n}{1 - \mathbb{E}_{y^*}[m_{n'}]} + O(|m|^2) \tag{OA9}$$

Substituting this into Equation OA6, we have

$$
\begin{aligned}
\vec{1}^T dY^1 &= \frac{1}{1-\mathbb{E}_{y^*}[m_n]} + \frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y^*}[m_n]}{1-\mathbb{E}_{y^*}[m_n]} \\
&+ \mathbb{E}_{\partial y^1}[m_n]\left(1 + \frac{\mathbb{E}_{\partial y^1}[m_n^{\text{next}}]}{1-\mathbb{E}_{y^*}[m_{n'}]} + O(|m|^2) - 1 - \frac{\mathbb{E}_{\partial y^1}[m_n]}{1-\mathbb{E}_{y^*}[m_n]}\right) \\
&+ \mathbb{C}\text{ov}_{\partial y^1}\left[m_n, 1 + \frac{m_n^{\text{next}}}{1-\mathbb{E}_{y^*}[m_{n'}]} + O(|m|^2)\right] \\
&= \frac{1}{1-\mathbb{E}_{y^*}[m_n]} + \frac{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y^*}[m_n]}{1-\mathbb{E}_{y^*}[m_n]} + \frac{\mathbb{E}_{\partial y^1}[m_n]}{1-\mathbb{E}_{y^*}[m_n]}\mathbb{E}_{\partial y^1}[m_n^{\text{next}} - m_n] \\
&+ \left(\frac{1}{1-\mathbb{E}_{y^*}[m_n]}\right)\mathbb{C}\text{ov}_{\partial y^1}\left[m_n, m_n^{\text{next}}\right] + O(|m|^3)
\end{aligned}
\tag{OA10}
$$

Rearranging yields Equation 18. □

A.4. Proof of Proposition 3

The full version of the planner's problem, Equation 20, is

$$
\begin{aligned}
\max_{\{c_{ni}^t, l_n^t, Q_i^t, G_i^t, \tau_n^t\}_{t\in\{1,2\}, n\in N, i\in\mathcal{I}^t}} \quad & W \equiv \sum_{n\in N}\mu_n\lambda_n\sum_{t=1,2}\beta_n^{t-1}\left[u_n^t(\tilde{c}^1) - v_n^t(\tilde{l}^t) + w_n^t(G^t)\right] \\
\text{s.t. } & (c_n^1, c_n^2, l_n^2) \text{ solves Equation 19 given } l_n^1 \\
& Q^t = \mu^T c^t + \widehat{X}^t(z^t)Q^t + G^t \\
& \widehat{\mu} l^1 = l^1(\widehat{L}^1 Q^1),\ \mu^T l^2 = \vec{1}^T\widehat{L}^2(z^t)Q^2 \\
& \vec{1} \equiv p^t = \left(I - \widehat{X}^t(z^t)\right)^{-1}\widehat{L}^t(z^t)\vec{1} \\
& \vec{1}^T G^1 + \frac{\vec{1}^T G^2}{1+r^1} + \mu^T\tau^1 + \frac{\mu^T\tau^2}{1+r^1} = 0
\end{aligned}
\tag{OA11}
$$

Proof. To begin, we define κ_n^t to be n's marginal value of additional expenditure in period t, i.e. for all i, $u_{nc_i}^t = \kappa_n^t$ (recall prices are normalized to one). Therefore,

$$
\begin{aligned}
dW &= \sum_{n\in N}\lambda_n\mu_n\sum_{t=1,2}\beta_n^{t-1}\left(u_{nc}^t dc_n^t - v_n^{t\,\prime} dl_n^t + w_{nG}^t dG^t\right) \\
&= \sum_{n\in N}\lambda_n\mu_n\sum_{t=1,2}\beta_n^{t-1}\left[\kappa_n^t\left(\vec{1}^T dc_n^t - \frac{v_n^{t\,\prime}}{\kappa_n^t}dl_n^t\right) + w_{nG}^t dG^t\right]
\end{aligned}
\tag{OA12}
$$

Next note that in the second period, free labor supply implies $v_n^{2\prime} = \kappa_n^2$. In the first, there may be some wedge Δ_n such that $v_n^{1\prime} = \kappa_n^1(1+\Delta_n)$; a positive wedge indicates that n works as if the wage was higher than it is, i.e. oversupplies labor; a negative wedge represents

involuntary un(der)employment. In these terms, we have

$$dW = \sum_{n \in N} \lambda_n \kappa_n^1 \mu_n \left[-\Delta_n dl_n^1 + \sum_{t=1,2} \frac{\kappa_n^t}{\kappa_n^1} \beta_n^{t-1} \left(\vec{1}^T dc_n^t - dl_n^t \right) + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \frac{\beta_n w_{nG}^2}{\kappa_n^1} dG^2 \right) \right] \tag{OA13}$$

Next, define $\widetilde{\lambda}_n = \lambda_n \kappa_n^1$. Also note that $\frac{\kappa_n^t}{\kappa_n^1} \beta_n^{t-1} = 1$ for $t = 1$. For $t = 2$, we use the modified Euler equation:

$$\kappa_n^1 = \beta_n \frac{1 + r^1}{1 - \phi_n} \kappa_n^2 \tag{OA14}$$

where ϕ_n is a borrowing wedge. $\phi_n \geqslant 0$ is positive when households behave as if interest rates are higher than in reality, i.e. consume more in the future than they would like; this corresponds to borrowing constraints. This gives us

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[-\Delta_n dl_n^1 + \left(\vec{1}^T dc_n^1 - dl_n^1 \right) + \frac{1 - \phi_n}{1 + r^1} \left(\vec{1}^T dc_n^2 - dl_n^2 \right) + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left(\frac{1 - \phi_n}{1 + r^1} \right) \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \right] \tag{OA15}$$

Differentiating the household's lifetime budget constraint (at constant r^1):

$$\vec{1}^T dc_n^1 - dl_n^1 + \frac{\vec{1}^T dc_n^2 - dl_n^2}{1 + r^1} = -d\tau_n^1 - \frac{d\tau_n^2}{1 + r^1} \tag{OA16}$$

Plugging this in, we have:

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[-\Delta_n dl_n^1 + \phi_n \left(\vec{1}^T dc_n^1 - dl_n^1 \right) - (1 - \phi_n) \left(d\tau_n^1 + \frac{d\tau_n^2}{1 + r^1} \right) + \left(\frac{w_{nG}^1}{\kappa_n^1} dG^1 + \left(\frac{1 - \phi_n}{1 + r^1} \right) \frac{w_{nG}^2}{\kappa_n^2} dG^2 \right) \right] \tag{OA17}$$

For households with non-strictly-binding borrowing constraints, $\phi_n = 0$. For households with $\phi_n > 0$, the borrowing constraint binds:

$$\underline{s}_n^1 = l_n^1 - \tau_n^1 - \vec{1}^T c_n^1 \implies \vec{1}^T dc_n^1 - dl_n^1 = -d\tau_n^1 \tag{OA18}$$

Defining the within-period willingness to pay for government expenditure as $WTP_n^t = \frac{w_{nG}^t}{\kappa_n^t}$, we arrive at the final expression:

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[-\Delta_n dl_n^1 - \left(d\tau_n^1 + (1 - \phi_n) \frac{d\tau_n^2}{1 + r^1} \right) + \left(WTP_n^1 dG^1 + (1 - \phi_n) \frac{WTP_n^2}{1 + r^1} dG^2 \right) \right] \tag{OA19}$$

□

A.5. Proof of Proposition 4

Proof. The proof of this result relies on material in Online Appendix B.6 on characterizing optimal fiscal policy; consult this section and the results therein before proceeding with this proof.

We first prove the result for first-period transfers. At any optimum, we know that

Equation OA86 must hold for all policy variations $\tau_\varepsilon^1 \in \mathbb{R}^N$ that only vary first-period transfers, keeping other instruments fixed. Taking $\tau_\varepsilon^1 = e_n$, the nth basis vector, we see that:

$$\left(\widetilde{\lambda}^T - \gamma\vec{1}\right)_n^T = \left(\widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C_{y^1}^1 R^1\right)^{-1} C_{y^1}^1\right)_n \tag{OA20}$$

Stacking these equations over n, we obtain:

$$\left(\widetilde{\lambda} - \gamma\vec{1}\right)^T = \widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C_{y^1}^1 R^1\right)^{-1} C_{y^1}^1 \tag{OA21}$$

Since $\{e_n\}$ is a basis and Equation OA86 is linear, this equation fully encompasses the optimality condition of Proposition 13 with respect to first period transfers.

We can simplify this system of equations. First, see that:

$$R^1(I - C_{y^1}^1 R^1)^{-1} C_{y^1}^1 = \sum_{k=0}^{\infty} R^1 (C_{y^1}^1 R^1)^k C_{y^1}^1 = \sum_{k=1}^{\infty} R^1 C_{y^1}^1 \tag{OA22}$$

Adding $\widetilde{\lambda}^T \hat{\Delta}$ to both sides of Equation OA21, we therefore obtain:

$$\left(\widetilde{\lambda}(1+\widehat{\Delta}) - \gamma\vec{1}\right)^T = \widetilde{\lambda}^T \widehat{\Delta} \left(I - R^1 C_{y^1}^1\right)^{-1} \implies \left(\widetilde{\lambda}(1+\widehat{\Delta}) - \gamma\vec{1}\right)^T \left(I - R^1 C_{y^1}^1\right) = \widetilde{\lambda}^T \widehat{\Delta} \tag{OA23}$$

Now, express $R^1 C_{y^1}^1 = R^1 \overline{C}_{y^1}^1 \hat{m}$. Recognizing that all columns of the spending-to-income matrix $R^1 \overline{C}_{y^1}^1$ sum to one, as total spending is equal to total factor income, and—by assumption—that $\widetilde{\lambda}_n(1+\Delta_n)$ is constant across all households n except for those for which the nth row of $R^1 C_{y^1}^1$ is zero, (OA23) can be rewritten as:

$$\left(\widetilde{\lambda}(1+\widehat{\Delta}) - \gamma\vec{1}\right)^T (I - \hat{m}) = \widetilde{\lambda}^T \widehat{\Delta} \tag{OA24}$$

We therefore have all, for all n, that

$$\widetilde{\lambda}_n(1+\Delta_n) - \gamma = \frac{1}{1-m_n}\widetilde{\lambda}_n \quad \implies \quad \gamma = \widetilde{\lambda}_n \left(1 + \frac{m_n}{1+m_n}(-\Delta_n)\right) \tag{OA25}$$

We prove the result for first-period government spending in an analogous way. To begin, consider Equation OA86 for policy variations $G_\varepsilon^1 \in \mathbb{R}^{\mathcal{I}^1}$ that only vary first period expenditure. Again considering each basis vector of $\mathbb{R}^{\mathcal{I}^1}$ and stacking we obtain:

$$0 = \widetilde{\lambda}^T W T P^1 - (\gamma\vec{1}^T + \widetilde{\lambda}^T \widehat{\Delta} R^1) - \widetilde{\lambda}^T \widehat{\Delta} R^1 (I - C_{y^1}^1 R^1)^{-1} C_{y^1}^1 R^1 \tag{OA26}$$

This can be rewritten as:

$$\widetilde{\lambda}^T W T P^1 - \gamma\vec{1}^T = \widetilde{\lambda}^T R^1 \widehat{\Delta} (I - C_{y^1}^1 R^1)^{-1} \tag{OA27}$$

From the assumption that the social gains from government expenditure equal $\tilde{v}$, we have

that $\widetilde{\lambda}^T WTP^1 = \tilde{v}$. Moreover, by definition $\widetilde{\lambda\Delta} = \widetilde{\lambda}\widehat{\Delta}R$. Hence (OA27) can be rewritten as

$$\tilde{v}\vec{1}^T - \gamma\vec{1}^T = \widetilde{\lambda\Delta}^T \left(I - C^1_{y^1}R^1\right)^{-1} \tag{OA28}$$

Next, define $\widetilde{m}_i \equiv \left(m^T R^1\right)_i$ to be the rationing-weighted average MPC in the production of good i and let $\widehat{\widetilde{m}}$ be the corresponding matrix with $\widetilde{m}$ on the diagonal. Moreover, define $\widetilde{C}_{ji} \equiv (C^1_{y^1}R^1)_{ji}/\widetilde{m}_i$ to be the average direction of consumption of workers producing i, weighted by their MPC and marginal rationing in i's production.[41] Crucially, note that $\widetilde{C}\widehat{\widetilde{m}} = C^1_{y^1}R^1$ by construction and that $\vec{1}^T\widetilde{C}\widehat{\widetilde{m}} = \vec{1}^T\widehat{\widetilde{m}}$:

$$\vec{1}^T\widetilde{C}\widehat{\widetilde{m}} = \vec{1}^T C^1_{y^1}R^1 = m^T R^1 = \widehat{\widetilde{m}}^T \tag{OA29}$$

The first order condition for expenditures (OA28) is therefore equivalent to:

$$(\tilde{v} - \gamma)\vec{1}^T \left(I - C^1_{y^1}R^1\right) = (\tilde{v} - \gamma)\vec{1}^T \left(I - \widehat{\widetilde{m}}\right) = \widetilde{\lambda\Delta}^T \iff \gamma = \tilde{v} + \frac{1}{1 - \widetilde{m}_i}(-\widetilde{\lambda\Delta}_i) \quad \forall i \in \mathcal{I}^1. \tag{OA30}$$

□

A.6. Proof of Proposition 5

Proof. Under the proposition's assumptions, Equation 21 reduces to:

$$dW = \mu^T dl^1 - \mu^T d\tau^1 - \frac{\mu^T d\tau^2}{1 + r^1} \tag{OA31}$$

Moreover, by Equation 22, we have that:

$$\widehat{\mu}dl^1 = R^1 \left(I - C^1_{y^1}R^1\right)^{-1} \left(dG^1 - C^1_{y^1}\left(\widehat{\mu}d\tau^1 + \frac{\widehat{\mu}d\tau^2}{1 + r^1}\right)\right) \tag{OA32}$$

Combining these equations and rearranging:

$$\begin{aligned} dW &= \vec{1}^T R^1 \left(I - C^1_{y^1}R^1\right)^{-1} \left(dG^1 - C^1_{y^1}\left(\widehat{\mu}d\tau^1 + \frac{\widehat{\mu}d\tau^2}{1 + r^1}\right)\right) - \mu^T d\tau^1 - \frac{\mu^T d\tau^2}{1 + r^1} \\ &= \vec{1}^T \left(I - C^1_{y^1}R^1\right)^{-1} dG^1 + \vec{1}^T \left[\left(I - R^1 C^1_{y^1}\right)^{-1} R^1 C^1_{y^1} + I\right]\left(-\widehat{\mu}d\tau^1 - \frac{\widehat{\mu}d\tau^2}{1 + r^1}\right) \\ &= \vec{1}^T \underbrace{\left(I - C^1_{y^1}R^1\right)^{-1}}_{=dY^1/dG^1} dG^1 + \vec{1}^T \underbrace{\left(I - R^1 C^1_{y^1}\right)^{-1}}_{=dl^1/dy^1}\left(-\widehat{\mu}d\tau^1 - \frac{\widehat{\mu}d\tau^2}{1 + r^1}\right) \end{aligned} \tag{OA33}$$

Finally, one may add terms proportional to $\frac{dY^1}{dG^2} = 0$. □

[41] For any i with $\widetilde{m}_i = 0$, define $\widetilde{C}_{ji}$ in any way satisfying $\sum_j \widetilde{C}_{ji} = 1$.

A.7. Proof of Corollary 1

Proof. We first show that, if the bias and homophily effects are zero for all output and transfer shocks relative to some baseline income incidence y^*, then either $m_n = 0$ or $m_n^{\text{next}} = \mathbb{E}_{y^*}[m_{n'}]$. We then use this fact to obtain the conclusion of the corollary.

To start, fixing a single type $n \in N$, consider the bias term corresponding to a transfer shock with direct incidence $\partial y^1 = \hat{e}_n$ (i.e. only transfering to n).

$$\text{bias}_{\partial y^1}^{y^*} = \mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{\text{next}}] - \mathbb{E}_{y^*}[m_{n'}]\right) = m_n\left(m_n^{\text{next}} - \mathbb{E}_{y^*}[m_{n'}]\right) \tag{OA34}$$

The assumption that this is zero then implies that either $m_n = 0$ or $m_n^{\text{next}} = \mathbb{E}_{y^*}[m_{n'}]$.

To apply this fact, recall the definition $m_n^{\text{next}} = m^T R^1 \overline{C}_{y^1}^1$, where $\overline{C}_{y^1}^1$ is the normalized matrix of spending directions, i.e. $C_{y_1}^1 = \overline{C}_{y^1}^1 \widehat{m}$. Our previous observation—that for all n, $m_n = 0$ or $m_n^{\text{next}} = \mathbb{E}_{y^*}[m_{n'}]$—then implies that $m^T R^1 C_{y^1}^1 = (\vec{m}^{\text{next}})^T \widehat{m} = \mathbb{E}_{y^*}[m_{n'}] \cdot m^T$.

Applying this fact to the multipliers in Equation 25, we have

$$\begin{aligned}\vec{1}^T \frac{dY^1}{dG^1} &= \vec{1}^T \left(I - C_{y^1}^1 R^1\right)^{-1} = \sum_{k=0}^{\infty} \vec{1}^T \left(C_{y^1}^1 R^1\right)^k = \vec{1}^T + \overbrace{\vec{1}^T C_{y^1}^1}^{=m^T} R^1 + \sum_{k=1}^{\infty} \overbrace{\vec{1}^T C_{y^1}^1}^{=m^T} \left(R^1 C_{y^1}^1\right)^k R^1 \\ &= \vec{1}^T + m^T R^1 + \sum_{k=1}^{\infty} \mathbb{E}_{y^*}[m_n]^k m^T R^1 = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} m\right)^T R^1\end{aligned} \tag{OA35}$$

Moreover, we have that:

$$\begin{aligned}\vec{1}^T \frac{dl^1}{dy^1} &= \vec{1}^T \left(I - R^1 C_{y^1}^1\right)^{-1} = \vec{1}^T + \overbrace{\vec{1}^T R^1 C_{y^1}^1}^{=m^T} + \sum_{k=1}^{\infty} \overbrace{\vec{1}^T R^1 C_{y^1}^1}^{=m^T} \left(R^1 C_{y^1}^1\right)^k \\ &= \vec{1}^T + \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} m^T = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y^*}[m_n]} m\right)^T\end{aligned} \tag{OA36}$$

Substituting (OA35) and (OA36) into Equation 25 completes the proof. □

B. Additional Results and Extensions

Here we provide results on properties (including existence) of rationing equilibrium (B.1), derive the multiplier with interest rate effects (B.2), allow for imperfect competition with fixed markups (B.3), generalize our decomposition results to account also for supply shocks (B.4), analyze benchmark cases in which the network adjustments to the Keynesian multiplier are zero (B.5), provide first order conditions for optimal policy (B.6), and consider policy in the environment with imperfect competition (B.7). In the Supplementary Material, we extend the baseline model to many periods (allowing for an infinite horizon) (A.1), study the structure of the multiplier in a more canonical flexible-wage equilibrium (A.2), and provide a network reinterpretation of the multiplier at the zero lower bound (A.3).

B.1. Equilibrium Properties

In this Appendix, we ensure our analysis of the multiplier is well-posed and eliminate any nuisance terms that unnecessarily complicate the analysis. To this end, we first provide a no-substitution theorem that ensures prices are technologically determined – and thus independent of demand – and, second, prove the existence of a rationing equilibrium.

The following technical conditions on production technologies and household preferences are sufficient for the no-substitution theorem. Assumption 3 provides basic technical conditions on production and Assumption 4 imposes a simple positivity condition on demand such that there is demand for all goods.

Assumption 3. *For all i and z_i, production $F(X_i, L_i, z_i)$ is continuous, weakly increasing, strictly quasi-concave, and homogeneous of degree one in (X_i, L_i). Further, labor is essential in production, i.e. $F(X_i, 0, z_i) = 0$, and production is strictly increasing in labor. Finally, there exists some $\overline{p} \in \mathbb{R}_+^{\mathcal{I}^t}$ and $\{X_i, L_i\}_{i \in \mathcal{I}^t}$ s.t. for all i, $F(X_i, L_i, z_i) \geqslant 1$ and $\overline{p}X_i + L_i \leqslant \overline{p}_i$.*[42]

Assumption 4. *For any $\varrho, y^1, \tau, \theta$: for each good i, some household type n has $c_{ni}^t > 0$.*

Under these two rather weak assumptions, we can show that:

Proposition 6. *Under Assumptions 3 and 4, for a given z^t, there exists a unique p^t consistent with rationing equilibrium, independent of demand.*

Proof. We follow closely the proof technique used in Acemoglu and Azar (2020). We will prove the result for an economy with arbitrary time horizon for maximum applicability. Fix a time period t vector of productivity parameters z. For each i, define the unit cost function:

$$\kappa_i(p) = \min_{F(X_i, L_i, z_i) \geqslant 1,\ X_i, L_i \geqslant 0} pX_i + L_i \tag{OA37}$$

The minimum is well-defined owing to Assumption 3, which states that F is strictly increasing in labor, CRS, and strictly quasiconcave.

We now establish properties of the unit cost function on the domain $p \in \mathbb{R}_+^{\mathcal{I}}$. First, since labor is necessary for production, $\kappa_i(0) > 0$ for all i. Second, by the last part of Assumption 3, there exists $\overline{p}$ such that $\kappa_i(\overline{p}) \leqslant \overline{p}_i$ for all i. Finally, $\kappa_i(p)$ is weakly increasing in p by inspection. These three properties establish that $\kappa(p) \equiv (\kappa_1(p), ..., \kappa_{\mathcal{I}}(p))$ maps $\mathbb{O} \equiv \times_{i=1}^{\mathcal{I}} [0, \overline{p}_i] \to \mathbb{O}$ and is weakly increasing. Moreover, $\mathbb{O}$ is a complete lattice with respect to the following operators:

$$p \wedge q \equiv \big(min(p_1, q_1), ..., min(p_{\mathcal{I}}, q_{\mathcal{I}})\big) \quad \text{and} \quad p \vee q \equiv \big(max(p_1, q_1), ..., max(p_{\mathcal{I}}, q_{\mathcal{I}})\big) \tag{OA38}$$

By Tarksi's fixed point theorem, the set of fixed points $\{p \in \mathbb{R}_+^{\mathcal{I}} \mid \kappa(p) = p\}$ is therefore a complete lattice.

In order for p to be consistent with either our flexible-wage or rationing equilibrium, all operating firms must make zero profits. Assumption 4 implies that all firms operate in equilibrium, so $p = \kappa(p)$ is a necessary condition for any equilibrium. It therefore remains

[42] A sufficient but not necessary condition is that every good can be produced using only labor.

to show that κ has a unique fixed point. To this end, we first show that each κ_i is concave. For price vectors p and q and $\lambda \in (0,1)$, we construct the price vector:

$$p^\lambda = \lambda p + (1-\lambda)q \tag{OA39}$$

By cost minimization,

$$\kappa_i(p) \leqslant pX_i(p^\lambda) + L_i(p^\lambda) \qquad \text{and} \qquad \kappa_i(q) \leqslant qX_i(p^\lambda) + L_i(p^\lambda) \tag{OA40}$$

It follows that:

$$\kappa_i(p^\lambda) = p^\lambda X_i(p^\lambda) + L_i(p^\lambda) \geqslant \lambda\kappa_i(p) + (1-\lambda)\kappa_i(q) \tag{OA41}$$

establishing that each κ_i is a concave function.

Toward a contradiction, suppose κ has more than one fixed point. Then since the set of fixed points is a complete lattice, there must exist distinct fixed points p^*, p^{**} with $p_i^* \leqslant p_i^{**}$ for all i. Now take λ to be given by the following:

$$\lambda = \underset{i\in\mathcal{I}}{min}\ \frac{p_i^*}{p_i^{**}} \tag{OA42}$$

Note that $\lambda \in (0,1)$ since $p >> 0$ for all fixed points p, since $\kappa_i(0) > 0$ for all i and κ is weakly increasing. We have that $p_i^* \geqslant \lambda p_i^{**}$ for all $i \in \mathcal{I}$ with equality for at least one j by construction. For this j such that $p_j^* = \lambda p_j^{**}$, we then have

$$0 = \kappa_j(p^*) - p_j^* \geqslant \kappa_j(\lambda p^{**}) - \lambda p_j^{**} \geqslant (1-\lambda)\kappa_j(0) + \lambda\kappa_j(p^{**}) - \lambda p_j^{**} = (1-\lambda)\kappa_j(0) > 0 \tag{OA43}$$

where the first line follows from the zero profit condition, the second line follows from the fact that κ_i is weakly increasing and $\lambda \in (0,1)$, the third line follows from concavity of κ_i, the fourth line follows again from the zero profit condition, and the final line follows from positivity of costs. This is a contradiction. Hence, there must be a unique fixed point at all times t. This implies the stated result and also makes the no-substitution theorem applicable to Supplementary Appendix A.1 where we extend the baseline model to allow for multiple time periods. □

The existence of unique, positive prices $p^1(z^1), p^2(z^2) \in \mathbb{R}_+^{\mathcal{I}^t}$ consistent with equilibrium allows us to reduce the number of endogenous price variables in considering comparative statics that keep z^1 and z^2 fixed, allowing us to keep track of just the real interest rate. Implicit in this no-substitution economy is the assumption that good prices respond instantaneously to changes in technology, which is irrelevant in the case of demand shocks.

Moreover, combining Proposition 6 with constant returns to scale technology implies a simple form for aggregate input and labor demands. Formally:

Corollary 2. *The aggregate input demand $X^t(p^t, Q^t)$ and labor demand $L^t(p^t, Q^t)$ vectors are given by:*

$$X^t = \hat{X}(z^t)Q^t \quad L^t = \hat{L}(z^t)Q^t \tag{OA44}$$

where $\widehat{X}(z^t)$ is the matrix with i^{th} column $\widehat{X}_i(z^t)$ and $\widehat{L}(z^t)$ is the diagonal matrix with i^{th} entry $\widehat{L}_i(z^t)$.

Proof. Fixing z, by Proposition 6, there exists a unique price vector $p(z)$ consistent with equilibrium. The unit input demands for any firm i at this price solve the following program:

$$\left(\widehat{X}_i(z), \widehat{L}_i(z)\right) = \arg \min_{(X_i,L_i) \text{ s.t. } F(X_i,L_i,z_i)\geqslant 1} p(z)X_i + L_i \tag{OA45}$$

CRS then implies that for a firm producing Q_i units in equilibrium,

$$X_i = Q_i\widehat{X}_i(z) \quad L_i = Q_i\widehat{L}_i(z) \tag{OA46}$$

Stacking these equations over $\mathcal{I}^t$ gives

$$X^t = \hat{X}(z^t)Q^t \quad L^t = \hat{L}(z^t)Q^t \tag{OA47}$$

□

Proposition 6 implies two additional, useful results. First, the Leontief-inverse matrix always exists. Second, one can use the Leontief-inverse to obtain a useful closed-form expression for the demand-independent prices. This is stated formally in the following corollary:

Corollary 3. *The Leontief-inverse matrix $(I - \hat{X}(z))^{-1}$ exists. Moreover, prices are given uniquely by the following expression:*

$$p(z) = (I - \hat{X}(z)^T)^{-1}\hat{L}(z)\vec{1} \tag{OA48}$$

Proof. We first prove that the matrix $(I - \widehat{X}(z))$ is invertible. The zero-profit condition for all i implies that:

$$p(z)X_i + L_i = p_i(z)Q_i \tag{OA49}$$

Normalizing by the quantity yields:

$$p(z)\widehat{X}_i(z) + \widehat{L}_i(z) = p_i(z) \tag{OA50}$$

Stacking this equation yields the matrix equation:

$$\widehat{L}(z)\vec{1} + \widehat{X}^T(z)p(z) = p(z) \tag{OA51}$$

This allows us to solve for the unit labor demands as the unique diagonal matrix such that:

$$\widehat{L}(z)\vec{1} = \left(I - \widehat{X}(z)^T\right)p(z) \tag{OA52}$$

Iterating this equation $k \in \mathbb{N}$ times yields:

$$p(z) = \left(1 + \widehat{X}(z)^T + ... + \left(\widehat{X}(z)^T\right)^k\right)\ \widehat{L}(z)\vec{1} + \left(\widehat{X}(z)^T\right)^{k+1}p(z) \tag{OA53}$$

Recall that $\widehat{X}(z)$ is non-negative, $\widehat{L}(z)\vec{1}$ is strictly positive because labor is essential, and $p(z)$ is positive. A necessary condition for $p(z)$ to exist is therefore that $\left(\widehat{X}(z)^T\right)^k \to 0$ as $k \to \infty$. This implies that $\widehat{X}(z)^T$ (and therefore also $\widehat{X}(z)$) has modulus strictly less than unity. It is immediate that the inverse $\left(I - \widehat{X}(z)\right)^{-1}$ exists. (OA52) then implies the result. □

Throughout the paper we will write $\widehat{X}^t, \widehat{L}^t$ for $\widehat{X}(z^t), \widehat{L}(z^t)$ when z^t is fixed. We write $\widehat{X}$ and $\widehat{L}$ for the block-diagonal matrices composed of $\widehat{X}^1$ and $\widehat{X}^2$, and $\widehat{L}^1$ and $\widehat{L}^2$ respectively.

We now proceed to establish that the analysis of equilibrium is well posed by providing regularity conditions under which equilibria exist. To this end, we assume basic continuity properties of demand and that household consumption in the first period is bounded away from fully consuming first period income as income grows large.

Assumption 5. *The primitives satisfy the following properties:*

1. *The consumption and labor functions c_n^t and l_n^1 are continuous in r^1 and y^1.*
2. *For all $n, \varrho, \tau_n, \theta_n$, $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n)$ is weakly increasing in y_n^1.*
3. *For any p, τ, θ: there exists $\overline{y} \in \mathbb{R}_+$ and $\overline{c} < 1$ such that for all $n \in N$, $r^1 \in [\underline{r}, \overline{r}]$, and $y_n^1 > \overline{y}$, we have that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n) \leqslant \overline{c} y_n^1$.*
4. *Interest rates have an upper and lower bound, i.e. $r^1(Q) \in [\underline{r}, \overline{r}]$ and r is differentiable.*

This assumption is extremely mild and satisfied by virtually all standard household problems of which we are aware.[43] With this additional structure we are now able to prove the existence of rationing equilibria for the economy under consideration.

Proposition 7. *Under assumptions 3, 4, and 5, there exists a rationing equilibrium.*

Proof. Fix all exogenous parameters. Note that by Proposition 6, prices p^1 and p^2 are pinned down by technology and so can be taken as given as well.

The outline of the proof is as follows. First, for any interest rate r^1, we construct a function Ψ_{r^1} that maps vectors of first-period income to vectors of first-period income and show that any fixed point of this map corresponds to an equilibrium with constant r^1. Second, we extend this map to construct a second function Ψ that takes as inputs both a vector of incomes and an interest rate, and we show that any fixed point of this extended map corresponds to an equilibrium of the model. We then apply Brouwer's fixed point theorem to Ψ to show that such a fixed point exists.

First, by Assumption 5 we have the following two facts:

1. For any p^1, p^2, τ, θ: $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n)$ is weakly increasing in y_n^1 for any n, $r^1 \in [\underline{r}, \overline{r}]$
2. For any p^1, p^2, τ, θ: there exists some $\overline{y} \in \mathbb{R}_+$ and some $\overline{c} < 1$ such that $p^1 c_n^1(\varrho, y_n^1, \tau_n, \theta_n) \leqslant \overline{c} y_n^1$ for all n, $y_n^1 > \overline{y}$, $r^1 \in [\underline{r}, \overline{r}]$

Thus, for any vector of incomes y^1, first period consumption spending $p^1 C^1$ is bounded above:

$$p^1 C^1 \leqslant \overline{c}\overline{y} + \overline{c}\vec{1}' y^1 \tag{OA54}$$

Thus, aggregate spending is bounded above by:

$$p^1 C^1 + p^1 G^1 \leqslant \overline{c}(\overline{y} + \vec{1}' y^1) + \max_{r \in [\underline{r}, \overline{r}]} p^1 G^1(p^1, p^2, r^1, \tau, \theta_G) \tag{OA55}$$

[43] It is easy to see how Assumption 5 holds if households are utility maximizers whose utility functions satisfy various standard assumptions. Existence and continuity of the consumption and labor functions follow from continuity and quasiconcavity of utility, and from Berge's theorem. Satisfying the lifetime budget constraint follows from non-satiation. Consumption being asymptotically bounded away from first-period income follows from sufficiently decreasing marginal utility.

where this maximum exists by continuity of $G^1(\cdot)$ in r^1 and compactness of $[\underline{r}, \overline{r}]$. Since $\overline{c} < 1$, it follows that there exists $\overline{Y}$ such that if $y^1 \in Y^1 \equiv \{y^1 \in \mathbb{R}^N_+ \mid \vec{1}'y^1 \leqslant \overline{Y}\}$, then aggregate spending—and so, as all spending flows to wages, also the resulting aggregate income—is weakly less than $\overline{Y}$. Formally:

$$\forall r^1 \in [\underline{r}, \overline{r}], y^1 \in Y^1 : l^1 \left(\widehat{L}^1 (1 - \widehat{X}^1)^{-1} \left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right) \right) \in Y^1 \tag{OA56}$$

This observation allows us to define, for any $r^1 \in [\underline{r}, \overline{r}]$, a function $\Psi_{r^1} : Y^1 \to Y^1$ given by:

$$\Psi_{r^1}(y^1) = l^1 \left(\widehat{L}^1 (1 - \widehat{X}^1)^{-1} \left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right) \right) \tag{OA57}$$

where recall ϱ denotes (p^1, p^2, r^1) and where the previous argument establishes that $\Psi_{r^1}(y^1)$ is indeed contained in Y^1. Moreover, continuity of $l^1(\cdot), C^1(\cdot)$ and $G^1(\cdot)$ establishes that Ψ_{r^1} is a continuous function.

Second, we define an extended function $\Psi : Y^1 \times [\underline{r}, \overline{r}] \to Y^1 \times [\underline{r}, \overline{r}]$ by setting:

$$\Psi(y^1, r^1) = \left(\Psi_{r^1}(y^1), r^1(Q) \right) \tag{OA58}$$

where $Q = (Q^1, Q^2)$ is given by $Q^t = (1 - \widehat{X}^1)^{-1} \left(C^1(\varrho, y^1, \tau, \theta) + G^1(\varrho, y^1, \tau, \theta_G) \right)$ and where $r^1(\cdot)$ is the monetary policy function, which recall selects an interest rate in $[\underline{r}, \overline{r}]$.

Third, we now claim that Ψ has a fixed point (y^1, r^1). This follows from Brouwer's theorem: $Y^1 \times [\underline{r}, \overline{r}]$ is a compact, convex domain, and Ψ is continuous because $l^1(\cdot)$ and $r^1(\cdot)$ are continuous, $c^t_n(\varrho, y^1_n, \tau_n, \theta_n)$ is continuous in y^1_n and r^1, and $G^t(\varrho, \tau, \theta_G)$ is continuous in r^1.

Finally, given a fixed point (y^1, r^1) of Ψ, we can construct a rationing equilibrium as follows: Let p^t be the no-substitution-theorem prices implied by z^t. Let c^t_n, l^2_n, and G^t be given by the relevant functions taking in prices p^t, real rate r^1, and incomes y^1. Let production in each period be:

$$Q^t = (I - \widehat{X}^t)^{-1}(G^t + C^t) \tag{OA59}$$

The definition of the consumption, labor supply, and government spending function ensure that household and government budget constraints hold. The construction of Q^t ensures that each goods market clears. Because (y^1, r^1) is a fixed point, first period income is consistent with the rationing function and the first period labor market clears; also because (y^1, r^1) is a fixed point, the interest rate $r^1 = r^1(Q)$ is consistent with central bank policy. Finally, the second period labor market clears by Walras' law. □

As we have established conditions under which an equilibrium exists, our analysis of equilibria going forward will be well-posed. While the fixed-point theorems we use are familiar, we employ a somewhat different strategy to usual existence proofs in (i) leveraging the structure of no-substitution and (ii) clearing markets intertemporally and then constructing intratemporal market clearing from the resulting fixed point interest rate. This provides a common structure to both rationing equilibrium and flexible-wage equilibrium (see Supplementary Appendix A.2) existence and may be useful to other authors proving equilibrium

existence in economies with labor rationing.

B.2. The Output Multiplier with Interest Rate Effects

In Section 3, we assumed with Assumption 1 that the output multiplier was comprised solely of income multiplier effects, either due to interest rates not responding to output or households not responding to interest rates. For completeness, we provide below a version of the multiplier in Proposition 1 that takes these effects into account.

Proposition 8. *There exists a matrix M such that for any small shock to parameters $\partial x \in Span\{d\theta_G, d\theta, d\tau, dz\}$, there exists a selection from the equilibrium set such that the general equilibrium response in output is given by:*

$$dY = M\partial Q \tag{OA60}$$

where ∂Q is the partial equilibrium change in production associated with ∂x stacked over time periods. Moreover, the matrix M is given by:

$$M = \left(I - D\left(I - \widehat{X}\right)^{-1}\right)^{-1}, \quad where \quad D = \begin{bmatrix} C^1_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^1} & \left(C^1_{r^1} + G^1_{r^1}\right) r^1_{Q^2} \\ C^2_{y^1} l^1_{L^1} \widehat{L}^1 + \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^1} & \left(C^2_{r^1} + G^2_{r^1}\right) r^1_{Q^2} \end{bmatrix} \tag{OA61}$$

Proof. The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 7, we know that for each ω_n there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit ω. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Toward a contradiction, suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at e: household budget balance, government budget balance or market clearing. But by Assumption 5, continuity of the fiscal rule, continuity of the interest rate rule and continuity of the rationing function, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contradiction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

$$dr^1 = r^1_{Q^1} dQ^1 + r^1_{Q^2} dQ^2 = r^1_Q dQ \tag{OA62}$$

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions. Our differentiability assumptions allow us to express

$$\begin{aligned} dQ \;&= \widehat{X} dQ + \widehat{X}_z dz Q + C_{\hat{p}} \hat{p}_z dz + C_{r^1} dr^1 + C_{y^1} dy^1 + C_\tau d\tau + C_\theta d\theta \\ &\quad + G_{\hat{p}} \hat{p}_z dz + G_{r^1} dr^1 + G_\tau d\tau + G_{\theta_G} d\theta_G \end{aligned} \tag{OA63}$$

Plugging in for dr^1 and $dy^1 = l^1_{L^1} \widehat{L}^1 dQ^1 + l^1_{L^1} d\widehat{L}^1 Q^1$

$$dQ = \widehat{X} dQ + C_{y^1} l^1_{L^1} \widehat{L}^1 dQ^1 + (C_{r^1} + G_{r^1}) r^1_Q dQ + \partial Q \tag{OA64}$$

where here $\partial Q = (C_{\hat{p}} + G_{\hat{p}})\hat{p}_z dz + \widehat{X}_z dz Q + C_{y^1} l^1_{L^1} \widehat{L}^1_z dz Q^1 + (C_\tau + G_\tau)d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G$. Recognizing that $dY = (I - \widehat{X})dQ$ and substituting completes the proof.

□

B.3. Imperfect Competition

In this section we show how to incorporate imperfect competition in the form of fixed markups on marginal costs. Now, instead of each sector being populated by a continuum of perfectly competitive firms, we suppose that for all $i \in \mathcal{I}^t$ there is a single monopolist producing each good, charging a fixed markup of m_i^t over their marginal cost and making (and distributing) profits π_i^t.[44] Despite this, we argue that a no substitution theorem still holds and we can obtain analogous multiplier formulae once we augment labor income rationing with profit rationing. To do this, we have to slightly modify Assumption 3:

Assumption 6. *For each t there exists some $\overline{p}^t \in \mathbb{R}_+^{\mathcal{I}^t}$ and $\{X_i^t, L_i^t\}_{i \in \mathcal{I}^t}$ such that for all i, $F(X_i^t, L_i^t, z_i^t) \geqslant 1$ and $(1 + m_i^t)(\overline{p}^t X_i^t + L_i^t) \leqslant \overline{p}_i^t$*

Under this modified assumption, we can state and prove the modified no-substitution theorem with markups:

Proposition 9. *Under Assumptions 6 and 4, for a given z^t and m^t, there exists a unique p^t consistent with both flexible-wage and rationing equilibrium, independent of demand.*

Proof. We modify the proof of proposition 6 to accommodate markups. Each firm now sets a price $p_i = (1 + m_i^t)\kappa_i(p)$, where κ_i is i's unit cost function. That is, i prices goods as though it were a competitive firm with production function $\frac{1}{1+m_i^t} F(X_i^t, L_i^t, z_i^t)$. Consider now a modified economy without markups and production functions given by the previously-stated markup-adjusted production functions. Assumption 6 implies that Assumption 3 holds in this modified economy. The result then follows by application of Proposition 6. □

We assume that profits from each firm are distributed to households according to an exogenous profit rationing function $\Pi^t : \mathbb{R}^{\mathcal{I}} \to \mathbb{R}^N$ satisfying $\sum_{i \in \mathcal{I}} \pi_i^t = \sum_{n \in N} \Pi^t(\pi^t)_n$ for all $\pi^t \in \mathbb{R}^{\mathcal{I}}$. We let $d_n^t = \Pi^t(\pi^t)_n$ represent household n's total dividend income in period t. With profits, household income is comprised of rationed first-period labor income, chosen second-period labor income, and (not chosen) dividend income in both periods. We therefore allow household consumption and labor supply functions to depend on d_n^t directly.

We can now state a profit-inclusive Keynesian cross. Note that the only difference to Proposition 8 comes from the need to account for changes in profits, how these are distributed to households as dividends, and their directed MPCs out of dividends.[45]

[44] One microfoundation for constant markups is that industries are comprised of a continuum of firms, with each other firm's and household's demands having the same CES aggregator for these firms' varieties.

[45] For the sake of generality, we distinguish between aggregate MPC out of dividend and labor income, i.e. $C_d^t \neq C_y^t$. Of course, for utility-maximizing households, these will be the same provided the income arrives in the same period.

Proposition 10. *For any small shock to parameters there exist a pair of rationing equilibria production Q and $Q+dQ$ before and after the shock. If the shock induces a partial equilibrium change in production ∂Q, the general equilibrium change dQ is given to first order by:*

$$dQ = \widehat{X}dQ + (C_r + G_r)r_Q dQ + C_y l^1_{L^1}\widehat{L}^1 dQ^1 + C_\pi \widehat{\Pi} dQ + \partial Q \tag{OA65}$$

where C_π is the matrix of household directed MPCs out of profit income, $\widehat{\Pi}$ is the block diagonal matrix composed of $\widehat{\Pi}^1$ and $\widehat{\Pi}^2$, and where $\widehat{\Pi}^t$ is the diagonal matrix with i^{th} entry $m^t_i p^t_i$, and all quantities are evaluated at the initial equilibrium.

Proof. This proof simply modifies the proof of Proposition 8. It is stated in full for clarity. The existence of two nearby equilibria is a consequence of the upper hemicontinuity of the equilibrium set in the parameters. Consider a sequence of parameters $\{\omega_n\}$ such that $\omega_n \to \omega$. By Proposition 7, we know that for each ω_n there exists a corresponding set of equilibria $\mathcal{E}_n$. Moreover let $\mathcal{E}(\omega)$ be the set of equilibria corresponding to the limit ω. Now consider an arbitrary sequence of equilibria $\{e_n\}$ such that $e_n \in \mathcal{E}_n$ for all $n \in \mathbb{N}$ and $e_n \to e$. Toward a contradiction, suppose that the set of equilibria is not UHC in the parameters, *i.e.* $e \notin \mathcal{E}(\omega)$. It follows that one of the following does not hold at e: household budget balance, government budget balance or market clearing. But by Assumption 5, continuity of the fiscal rule, continuity of the interest rate rule, continuity of the rationing function and continuity of the profit allocation function, we know that all functions in these expressions are continuous. It follows that there exists $m \in \mathbb{N}$ such that $e_m \notin \mathcal{E}_m$, a contradiction. This completes the proof that the equilibrium set is UHC.

Totally differentiating the interest rate rule, we can express the change in the real interest rate in terms of changes in demand:

$$dr^1 = r^1_{Q^1} dQ^1 + r^1_{Q^2} dQ^2 = r^1_Q dQ \tag{OA66}$$

Now, stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions:

$$\begin{aligned} dQ \;&= \widehat{X}dQ + \widehat{X}_z dzQ + C_{\hat{p}}\hat{p}_z dz + C_{r^1}dr^1 + C_{y^1}dy^1 + C_\tau d\tau + C_\theta d\theta \\ &\quad + G_{\hat{p}}\hat{p}_z dz + G_{r^1}dr^1 + G_\tau d\tau + G_{\theta_G}d\theta_G + C_\pi \hat{\Pi} dQ \end{aligned} \tag{OA67}$$

Plugging in for dr^1 and $dy^1 = l^1_{L^1}\widehat{L}^1 dQ^1 + l^1_{L^1} d\widehat{L}^1 Q^1$

$$dQ = \widehat{X}dQ + C_{y^1} l^1_{L^1}\widehat{L}^1 dQ^1 + (C_{r^1} + G_{r^1})r^1_Q dQ + C_\pi \hat{\Pi} dQ + \partial Q \tag{OA68}$$

where here $\partial Q = (C_{\hat{p}} + G_{\hat{p}})\hat{p}_z dz + \widehat{X}_z dzQ + C_{y^1} l^1_{L^1}\widehat{L}^1_z dzQ^1 + (C_\tau + G_\tau)d\tau + C_\theta d\theta + G_{\theta_G} d\theta_G$. □

B.4. Network Decompositions for Supply Shocks

We now derive network decompositions of the multiplier as in Section 3.2 that are valid for both demand and supply shocks, extending the earlier analysis. To this end, we see that

changes in GDP when we consider a supply shock have two distinct components:

$$d(GDP) \equiv d(p^{1T}Y^1) = \underbrace{p^{1T}dY^1}_{\text{Change in Product}} + \underbrace{dp^{1T}Y^1}_{\text{Change in Price Index}} \tag{OA69}$$

Where it is without loss to redefine units of consumption goods and evaluate at an initial equilibrium with $p^{1T} = \vec{1}$. Propositions 2 and 12 decompose the first term $\vec{1}^T dY^1$. To achieve our decomposition for supply shocks, we therefore need only compute $dp^{1T}Y^1$. To this end, we can employ Corollary 3, which shows prices are a closed-form function of z:

$$p^1(z) = \left(1 - \widehat{X}^1(z)^T\right)^{-1} \widehat{L}^1(z)\vec{1} \tag{OA70}$$

It follows that the change in GDP can then be decomposed as before but with a new term which depends only on the IO matrix and labor shares and not labor rationing or household consumption. This is stated formally below:

Proposition 11. *The total change in first-period output due to a shock with unit-magnitude labor income incidence ∂y^1 can be approximated as:*

$$\begin{aligned} d(p^{1T}Y^1) = & \frac{1}{1 - \mathbb{E}_{y*}[m_n]} \Bigg(1 + \underbrace{\mathbb{E}_{\partial y^1}[m_n] - \mathbb{E}_{y*}[m_n]}_{\textit{Incidence effect}} + \underbrace{\mathbb{E}_{\partial y^1}[m_n]\left(\mathbb{E}_{\partial y^1}[m_n^{next}] - \mathbb{E}_{y*}[m_n]\right)}_{\textit{Biased spending direction effect}} \\ & + \underbrace{\mathbb{C}ov_{\partial y^1}[m_n, m_n^{next}]}_{\textit{Homophily effect}} \Bigg) + \underbrace{d\left[\left(1 - \widehat{X}^1(z)^T\right)^{-1} \widehat{L}^1(z)\vec{1}\right]^T Y^1}_{\textit{Price Effect}} + O^3(|m|) \end{aligned} \tag{OA71}$$

where y^* is any reference income weighting of unit-magnitude and m^i_{next} is the average MPC of households who receive as income i's marginal dollar of spending.

Proof. Recall that we have:

$$d(p^{1T}Y^1) = \underbrace{p^{1T}dY^1}_{\text{Change in Product}} + \underbrace{dp^{1T}Y^1}_{\text{Change in Price Index}} \tag{OA72}$$

which we can always take as $d(p^{1T}Y^1) = \vec{1}^T dY^1 + dp^{1T}Y^1$ through an appropriate renormalization of the initial units of the goods.

By Proposition 2, $1^T dY^1$ consists of all terms in (OA71) except the price effect. We now need only compute the term $dp^{1T}Y^1$. To this end, from Corollary 3 we have that:

$$p^1(z) = \left(1 - \widehat{X}^1(z)^T\right)^{-1} \widehat{L}^1(z)\vec{1} \implies dp^{1T}Y^1 = \underbrace{d\left[\left(1 - \widehat{X}^1(z)^T\right)^{-1} \widehat{L}^1(z)\vec{1}\right]^T Y^1}_{\text{Price Effect}} \tag{OA73}$$

Adding the two terms yields the claimed expression and completes the proof. □

B.5. Special Cases Where Network Effects in Propagation Vanish

In the main text, we briefly discussed two important cases where network effects in shock propagation vanish. Here, we state and more formally discuss these results.

Proposition 12. *The following statements are true:*

1. *(No incidence or bias effects) Suppose that consumption preferences and labor rationing are homothetic, that no households are net borrowers in period 1, and that there is no government spending.*[46] *Then, for a GDP-proportional, unit-magnitude demand shock, the incidence and bias effects are zero, so that we have:*

$$\vec{1}^T dY^1 = \frac{1}{1-\mathbb{E}_{y^1}[m_n]}\left(1 \; + \; \underbrace{\mathbb{C}ov_{y^1}[m_n, m_n^{next}]}_{\textit{Homophily effect}}\right) + \; O^3(|m|) \tag{OA74}$$

   where y^1 is the vector of first-period incomes.
2. *(No incidence, bias, or homophily effects) Suppose that all industries have a common rationing-weighted average MPC, m.*[47] *Then the incidence, bias, and homophily effects are zero, so that for any reference weighting y^* that can be induced by a demand shock, the change in output corresponding to any unit-magnitude demand shock is:*[48]

$$\vec{1}^T dY^1 = \frac{1}{1-\mathbb{E}_{y^*}[m_n]} = \frac{1}{1-m} \tag{OA76}$$

Proof. We prove the two claims separately:

1. Recalling that $(m^{\text{next}})^T \equiv m^T\mathcal{G}$, and the shock satisfies $\partial y^1 \propto y^1$, the following are equivalent:

$$m^T\mathcal{G}y^1 - m^T y^1 = 0 \iff \mathbb{E}_{\partial y^1}[m_n^{\text{next}} - m_n] = 0 \tag{OA77}$$

   It therefore suffices to show that $\mathcal{G}y^1 = y^1$.

   Plugging in the definition of $\mathcal{G}$, we have $\mathcal{G}y^1 = l^1_{L^1}\widehat{L}^1\left(I - \widehat{X}^1\right)^{-1}\overline{C}^1_{y^1}y^1$. Since each household saves zero on net, y^1 is equal to total spending. Homotheticity of consumption implies that $\overline{C}^1_{y^1}y^1$, then, is the vector of total consumption of goods; since there is no government spending, this equals total output, Y^1. Finally, homotheticity of rationing implies that $l^1_{L^1}\widehat{L}^1\left(I - \widehat{X}^1\right)^{-1}Y^1 = y^1$.
2. Recall by Proposition 1 that when either $C_{r^1} + G_{r^1} = 0$ or $r^1_{Q^1} = 0$, the general equilibrium effect on income of a partial equilibrium shock is given by:

$$dY^1 = \left(I - C^1_y l^1_{L^1}\widehat{L}^1(1-\widehat{X}^1)^{-1}\right)^{-1}\partial Q^1 \tag{OA78}$$

[46] By homothetic labor rationing, we mean that marginal and average rationing of income are equal. Formally, if we let $\mathcal{L}^1 \equiv \widehat{L}^1\left(I - \widehat{X}^1\right)^{-1}Y^1$ be the vector of first-period firm-level labor bills, then we require that $y^1 = l^1_{L^1}\mathcal{L}^1$.

[47] Formally, $\sum_{n\in N}(l^1_{L^1})_{ni}m_n = m$ for all $i \in \mathcal{I}^1$.

[48] Formally, saying that y^* can be induced by a demand shock says that there exists a ∂Q^* such that:

$$y^* = l^1_{L^1}\widehat{L}^1\left(I - \widehat{X}^1\right)^{-1}\partial Q^* \tag{OA75}$$

We wish to investigate whether there exists some $m \in (0,1)$ such that the following holds for all ∂Q:

$$\vec{1}^T dY^1 = \frac{1}{1-m}\vec{1}^T \partial Q^1 \tag{OA79}$$

First, we note a simple fact of linear algebra. Suppose an invertible matrix M has columns summing to some constant m. This is equivalent to:

$$\vec{1}^T M v = m\vec{1}^T v, \quad \forall v \tag{OA80}$$

It is then true that for any v:

$$m\vec{1}^T(M^{-1}v) = \vec{1}^T M(M^{-1}v) = \vec{1}^T v \tag{OA81}$$

Thus, M^{-1} has columns summing to $\frac{1}{m}$.
Second, note that the desired result (OA79) holds if and only if

$$\left(I - C_y^1 l_{L^1}^1 \widehat{L}^1 (1-\widehat{X}^1)^{-1}\right)^{-1} \tag{OA82}$$

sums to $\frac{1}{1-m}$. This is equivalent, by the first observation, to the fact that each column of

$$C_y^1 l_{L^1}^1 \widehat{L}^1 (1-\widehat{X}^1)^{-1} \tag{OA83}$$

sums to m.
It remains to show that this claim is equivalent to the condition provided in the statement of the Proposition. Namely, we must show that

$$\vec{1}^T C_y^1 l_{L^1}^1 \widehat{L}^1 (1-\widehat{X}^1)^{-1} = m\vec{1}^T \iff \vec{1}^T C_y^1 l_{L^1}^1 = m\vec{1}^T \tag{OA84}$$

Multiplying each side by $(I - \widehat{X}^1)(\widehat{L}^1)^{-1}$—which exists since labor is essential in production—reveals that (OA84) holds if $(I - \widehat{X}^1)(\widehat{L}^1)^{-1}$ has columns summing to one.
By our earlier linear algebra observation, this holds if and only if $\widehat{L}^1(1-\widehat{X}^1)^{-1}$ has columns summing to one. This can be seen by recalling the no-profit condition

$$p^1 = (I - (\widehat{X}^1)^T)^{-1}\widehat{L}^1\vec{1}, \tag{OA85}$$

using our normalization $p = \vec{1}$, and taking the transpose of both sides.

□

The first part of the proposition shows how, even in a "homothetic economy," heterogeneity in household consumption baskets and sectoral employment can generate network effects through homophily. This happens even at the same time as homotheticity eliminates the bias effect by ensuring that each household's marginal consumption is proportional to its initial consumption, so that the income-weighted average of marginal consumption is proportional to output. Still, when households with different MPCs direct their spending toward different goods, the households employed to produce the goods consumed by higher-MPC

households experience a greater change in income – not from the initial, uniform shock, but from the economy's response to it. Insofar as these households have different MPCs from the average, homophily is still possible. This mechanism generates non-neutrality for the multiplier, even if the economy and the shock considered are "neutral" in all other aspects. Aggregate neutrality requires (to second order in MPCs) that the economy feature exactly zero correlation between households' MPCs and the MPCs of the households they spend on.

The second part of the proposition imposes that each firm's marginal employees have the same average MPC as one another. This eliminates the incidence, bias, and homophily effects, leaving only the classical Keynesian multiplier. That is, wherever in the economy a shock strikes, and however it spreads through directed consumption and the IO network, the change in aggregate consumption generated by the reduction in firm revenue is the same. Of course, a particular special case that satisfies these conditions is when there is a single good and a single household (in which case $l^1_{L^1} = 1$). Note that even when the traditional Keynesian multiplier obtains, the aggregate MPC need not equal either the average MPC or the income-weighted MPC of the population; this is the case only when each firm's marginal employees have the population average MPC.

B.6. Optimal Policy at a Global Optimum

In the main text, we focused primarily on small changes in welfare corresponding to small changes in policy. In this section, we specialize to the case of small changes in policy *at an optimum*. Thus, the corresponding changes in welfare are second order.

Our first result decomposes the first-order condition for optimal government spending and transfers into five distinct mechanisms. This is closely related to Proposition 3 in the main text, which considers the change in welfare away from the global optimum.

Proposition 13. *Suppose taxes τ^{1*}, τ^{2*} and expenditures G^{1*}, G^{2*} solve the planner's problem. Now consider a change in policy $\tau^t = \tau^{t*} + \varepsilon\tau^t_\varepsilon, G^t = G^{t*} + \varepsilon G^t_\varepsilon$, indexed by ε. The following first-order condition holds:*

$$
\begin{aligned}
0 = & \underbrace{\left(\widetilde{\lambda}^T \widehat{\mu} WTP^1 - (\gamma \vec{1}^T + \widetilde{\lambda}^T \widehat{\Delta} R^1)\right) G^1_\varepsilon}_{\textit{Opportunistic government spending}} + \underbrace{\frac{\left(\widetilde{\lambda}^T \widehat{\mu}(I - \widehat{\phi}) WTP^2 - \gamma \vec{1}^T\right) G^2_\varepsilon}{1 + r^1}}_{\textit{Short-termist government spending}} \\
& \underbrace{- (\widetilde{\lambda} - \gamma \vec{1})^T \widehat{\mu} \left(\tau^1_\varepsilon + \frac{\tau^2_\varepsilon}{1 + r^1}\right)}_{\textit{Pure redistribution}} \underbrace{+ \widetilde{\lambda}^T \frac{\widehat{\phi}\widehat{\mu}\tau^2_\varepsilon}{1 + r^1}}_{\textit{Relaxation of borrowing constraints}} \\
& \underbrace{- \widetilde{\lambda}^T \widehat{\Delta} R^1 \left(I - C^1_{y^1} R^1\right)^{-1} C^1_{y^1} \left(R^1 G^1_\varepsilon - \widehat{\mu}\tau^1_\varepsilon - \frac{1_{\phi_n = 0} \widehat{\mu}\tau^2_\varepsilon}{1 + r^1}\right)}_{\textit{Keynesian stimulus (alleviation of involuntary unemployment)}}
\end{aligned}
\tag{OA86}
$$

where γ is the marginal value of public funds.

Proof. The planner takes prices and—locally—the interest rate as given. Goods and labor market clearing and first-period rationing determine the change in first-period employment

as a function of G^1_ε and τ^1_ε. We are left with the following first-order condition:

$$0 = dW + \gamma \left[\mu^T \tau^1_\varepsilon + \frac{\mu^T \tau^2_\varepsilon}{1+r^1} - \vec{1}^T G^1_\varepsilon - \frac{\vec{1}^T G^2_\varepsilon}{1+r^1} \right] \tag{OA87}$$

where dW is as in Equation 21. This gives an expression for the change in welfare in terms of τ_ε, G_ε, and l^1_ε, the change in first-period employment. By Equation 12, $\widehat{\mu}\ l^1_\varepsilon = R^1(I - C^1_{y^1} R^1)^{-1} \partial Q^1$, where $R^1 \equiv l^1_{L^1} \widehat{L}^1 \left(I - \widehat{X}^1 \right)^{-1}$ and $\partial Q^1 = G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \tau^1_\varepsilon - C^1_{y^2} \widehat{\mu} \tau^2_\varepsilon$. For borrowing-constrained households, $C^1_{y^2} = 0$; they would already like to substitute additional consumption toward the first period but are constrained not to do so. Other households are Ricardian, implying $C^1_{y^2} = \frac{C^1_{y^1}}{1+r^1}$. Plugging in for dW, and using matrix notation, we have

$$\begin{aligned} 0 = \widetilde{\lambda}^T \Bigg[& - \widehat{\Delta} R^1 (I - C^1_{y^1} R^1)^{-1} \left(G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n = 0}\ \tau^2_\varepsilon}{1+r^1} \right) \right) - \left(\widehat{\mu} \tau^1_\varepsilon + \frac{\widehat{\mu}(I - \widehat{\phi}) \tau^2_\varepsilon}{1+r^1} \right) \\ & + \left(\widehat{\mu} WTP^1 G^1_\varepsilon + \widehat{\mu}(I - \widehat{\phi}) \frac{WTP^2}{1+r^1} G^2_\varepsilon \right) \Bigg] + \gamma \left(\mu^T \tau^1_\varepsilon + \frac{\mu^T \tau^2_\varepsilon}{1+r^1} - \vec{1}^T G^1_\varepsilon - \frac{\vec{1}^T G^2_\varepsilon}{1+r^1} \right) \end{aligned} \tag{OA88}$$

Now, observe that the term on the first line can be rewritten:

$$\begin{aligned} & R^1 (I - C^1_{y^1} R^1)^{-1} \left(G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n = 0}\ \tau^2_\varepsilon}{1+r^1} \right) \right) = R^1 \left(\sum_{k=0}^{\infty} (C^1_{y^1} R^1)^k \right) \left(G^1_\varepsilon - C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n = 0}\ \tau^2_\varepsilon}{1+r^1} \right) \right) \\ = & \left(R^1 G^1_\varepsilon + \left(\sum_{k=0}^{\infty} (C^1_{y^1} R^1)^k \right) C^1_{y^1} R^1 G^1_\varepsilon \right) - R^1 \left(\sum_{k=0}^{\infty} (C^1_{y^1} R^1)^k \right) C^1_{y^1} \widehat{\mu} \left(\tau^1_\varepsilon + \frac{1_{\phi_n = 0}\ \tau^2_\varepsilon}{1+r^1} \right) \\ = & R^1 G^1_\varepsilon + R^1 \left(I - C^1_{y^1} R^1 \right)^{-1} C^1_{y^1} \left(R^1 G^1_\varepsilon - \widehat{\mu} \tau^1_\varepsilon - \widehat{\mu} \frac{1_{\phi_n = 0}\ \tau^2_\varepsilon}{1+r^1} \right) \end{aligned} \tag{OA89}$$

Substituting this back in and rearranging, we obtain Equation OA86. □

The opportunistic government spending term is as in Werning (2011) and Baqaee (2015). It augments the standard first-order condition for government spending with a labor-wedge term, reflecting the fact that the social cost of additional government purchases is lower than the market cost when they are produced using underemployed labor. The second term is also an augmented version of the standard expression for government spending—this time in the second period. The borrowing wedge reflects the fact that households with binding borrowing constraints implicitly discount the future at a higher-than-market rate; the planner must account for this when deciding whether to make purchases on their behalf.

The third term of Equation OA86 is a standard, pure redistribution term, weighing the private benefits of transfers against the social cost (the MVPF). The fourth term augments this, when there are borrowing constraints. In particular, taxes in the second period are less costly to borrowing-constrained households since they discount the future more heavily than the market rate indicates.

Finally, the last line captures the value of stimulus brought on by changes in income—those corresponding to pure income transfers via taxes and labor market income earned by

employees producing expenditures.[49]

B.7. Optimal policy with imperfect competition

In this section, we extend the optimal policy results of section 4 to the more general environment with constant, non-zero markups. As is section 4 we normalize prices p_i^t to one throughout, without loss of generality.

To highlight as clearly as possible the parallels to the case without profits, we make two important assumptions. First—although in the first period, profit-creation is uninternalized by households—we assume that the government incentivizes second-period profit-creation with Pigouvian subsidies funded lump-sum by shareholders.

Assumption 7. *There is an ad-valorem subsidy s_i^2 on the purchase of i (for consumption or production), set equal to the profit rate m_i^2. It is funded directly by an additional lump-sum, second-period tax $\hat{\tau}_n^2$ defined by $\mu_n \hat{\tau}_n^2 = \sum_{i \in I} \left(\widehat{\Pi}_{ni}^2 \Big/ \sum_{n' \in N} \widehat{\Pi}_{n'i}^2 \right) s_i^2 Q_i^2$.*

Second, we assume that the MPC out of future profits is zero. This is a rather weak assumption, as the MPC out of even *current* capital income is small empirically.

Assumption 8. *For all households n, $C_{\pi^2}^1 = 0$.*

B.7.1. Planner's problem

We begin by defining the household's problem. It is the same as Equation 19 in section 4.1, except that households now also receive profit income, so that the budget constraint becomes $\tilde{l}^1 + \pi_n^1 - p^1 \cdot \tilde{c}^1 - \tau_n^1 \geqslant \underline{s}_n^1$. Note that this microfoundation implies $C_y = C_\pi$. That is, additional income from rationed labor has the same effects on consumption as additional income from profits.

As in section 4, we study the policy problem of a planner at the zero lower bound. Formally, the planner's problem is the same as in Equation 20 except that household behavior solves Equation 19 with the profit-inclusive budget constraint and aggregate variables evolve according to Equation OA65 with $r_Q = 0$.

B.7.2. Policy changes away from the optimum

This section considers changes in welfare due to small changes in not-necessarily-optimal policies, as in section 4.2. The only difference now is the presence of profits.

With this setup in mind, we now consider the change in welfare induced by changes in transfers and government expenditure, analogously to Proposition 3.

Lemma 3. *Under assumptions 7 and 8, the change in welfare dW due to a small change in taxes and government expenditure—at a constant interest rate—can be expressed as:*

$$dW = \sum_{n \in N} \tilde{\lambda}_n \mu_n \left[-\Delta_n dl_n^1 + d\pi_n^1 - \left(d\tau_n^1 + (1-\phi_n) \frac{d\tau_n^2}{1+r^1} \right) + \left(WTP_n^1 dG^1 + (1-\phi_n) \frac{WTP_n^2}{1+r^1} dG^2 \right) \right] \tag{OA90}$$

[49] If second period expenditures are held constant, then the net income transfer is zero, i.e. this term operates solely through redistribution to different households (who may spend differently).

where $\widetilde{\lambda}_n$ is the value the planner places on the marginal transfer of first-period wealth to a household of type n, Δ_n and ϕ_n are n's implicit first-period labor wedge and borrowing wedge, and WTP_n^t is the vector of n's marginal willingness to pay for period t government expenditures on each good, in period t dollars. The changes in first-period employment and profits are in turn given by

$$\begin{aligned} \widehat{\mu} dl^1 &= l^1_{L^1} \widehat{L}^1 \left(1 - \widehat{X}^1\right)^{-1} dY^1, \qquad \widehat{\mu} d\pi^1 = \widehat{\Pi} \left(1 - \widehat{X}\right)^{-1} dY^1, \\ dY^1 &= \left(I - C^1_{y^1} \left(l^1_{L^1} \widehat{L}^1 + \widehat{\Pi}^1\right) \left(I - \widehat{X}^1\right)^{-1}\right)^{-1} \partial Q^1 \end{aligned} \tag{OA91}$$

Proof. We follow the same steps as the proof of Proposition 3 (see Online Appendix A.4) up to the substitution of the budget constraint, which now includes profits. With profits, differentiating the household's lifetime budget constraint (at constant r^1) gives:

$$p^1 dc_n^1 - dl_n^1 - d\pi_n^1 + \frac{p^1 dc_n^2 - dl_n^2}{1 + r^1} = -d\tau_n^1 + \frac{d\pi_n^2 - d\hat{\tau}_n^2 - d\tau_n^2}{1 + r^1} \tag{OA92}$$

Note that since $\sum_{n' \in N} \widehat{\Pi}^2_{n'i} = m_i^2 = s_i^2$:

$$d\hat{\tau}_n^2 = \frac{1}{\mu_n} \sum_{i \in I} \left(\widehat{\Pi}^2_{ni} \Big/ \sum_{n' \in N} \widehat{\Pi}^2_{n'i} \right) s_i^2 dQ_i^2 = \frac{1}{\mu_n} \widehat{\Pi}^2_{ni} dQ_i^2 = d\pi_n^2 \tag{OA93}$$

Substituting in the change in the differentiated budget constraint, we have:

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[-\Delta_n dl_n^1 + \phi_n \left(p^1 dc_n^1 - dl_n^1\right) + (1 - \phi_n) \left(d\pi_n^1 - d\tau_n^1 - \frac{d\tau_n^2}{1 + r^1} \right) + \left(\frac{w^1_{nG}}{\kappa_n^1} dG^1 + \left(\frac{1 - \phi_n}{1 + r^1} \right) \frac{w^2_{nG}}{\kappa_n^2} dG^2 \right) \right] \tag{OA94}$$

For households with non-strictly-binding borrowing constraints, $\phi_n = 0$. For households with $\phi_n > 0$, the borrowing constraint $\underline{s}_n^1 = l_n^1 + \pi_n^1 - \tau_n^1 - p^1 c_n^1$ implies $p^1 dc_n^1 + d\tau_n^1 = dl_n^1 + d\pi_n^1$. Defining the within-period willingnesses to pay $WTP_n^t = \frac{w^t_{nG}}{\kappa_n^t}$, we arrive at the final expression:

$$dW = \sum_{n \in N} \widetilde{\lambda}_n \mu_n \left[-\Delta_n dl_n^1 + \left(d\pi_n^1 - d\tau_n^1 - (1 - \phi_n) \frac{d\tau_n^2}{1 + r^1} \right) + \left(WTP_n^1 dG^1 + (1 - \phi_n) \frac{WTP_n^2}{1 + r^1} dG^2 \right) \right] \tag{OA95}$$

Finally, the expressions for $dl, d\pi, dY$ come from rearranging Equation OA65 under assumption 8 and using $dY = (1 - \widehat{X}) dQ$. □

Studying Equation OA90 reveals a key insight: Under assumptions 7 and 8, the change in welfare due to a change in taxes and expenditures is the same as in an *as-if* economy without profits but where share-holders supply labor with a wedge -1. This labor supply wedge corresponds to complete under-employment; share-holders—who experience no marginal disutility of holding shares—would continue to be willing to hold shares until profits-per-revenue reached zero. Just like labor suppliers, share-holders do not choose their income but rather take it as given. This *as-if* representation of profits as under-employed labor allows us to carry over all of the results from Section 4 with minimal alterations.

Proposition 14. *Under assumptions 2, 7, and 8, the welfare change from a change in expenditures is proportional to the resulting change in output, whereas the welfare change from a change in transfers is proportional to the resulting change in income. Formally,*

$$dW = \vec{1}^T \frac{dY^1}{dG} dG + \vec{1}^T \frac{d(l+\pi)^1}{dy^1} \left(-\widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1+r^1} \right) \tag{OA96}$$

where $\frac{dY^1}{dG^1} = (1 - C^1_{y^1} R^1)^{-1}$ *and* $\frac{dY^1}{dG^2} = 0$ *are first-period output multipliers and* $\frac{d(l+\pi)^1}{dy^1} = (1 - R^1 C^1_{y^1})^{-1}$ *is the first-period income multiplier; here* $R^1 = \left(l^1_{L^1} \widehat{L}^1 + \widehat{\Pi}^1 \right) \left(I - \widehat{X}^1 \right)^{-1}$. *Moreover, if relative to some income incidence* y^*, $m^{next}_n = \mathbb{E}_{y*}[m_{n'}]$ *for all* n, *where* $m^{next}_i \equiv \left(m R^1 C^1_{y^1} \widehat{m}^{-1} \right)_i$,[50] *then under assumptions 2, 7, and 8,*

$$dW = \left(\vec{1} + \frac{1}{1 - \mathbb{E}_{y*}[m_n]} m \right)^T \left(R^1 dG^1 - \widehat{\mu} d\tau^1 - \frac{\widehat{\mu} d\tau^2}{1+r^1} \right) \tag{OA97}$$

Proof. Reinterpret profit income as labor supply with wedge -1, as discussed above. The proof then follows from Online Appendices A.6 and A.7. □

Key here is that assumption 2's imposition that all marginal labor supplies have a labor supply wedge of -1 matches with the shareholders' implicit labor supply wedge of -1 – both are indifferent to supplying more of their factor. Thus, there is zero social cost to any marginal employment, so the optimal policy maximizes output. As without markups, the output-maximizing policy targets MPC when bias and homophily effects are absent.

B.7.3. First-order conditions for optimal policy

The same *as-if* representation of profits as under-employed labor also allows us to carry over results from section B.6 to the case of imperfect competition.

Proposition 15. *Suppose taxes* τ^{1*}, τ^{2*} *and expenditures* G^{1*}, G^{2*} *solve the planner's problem. Now consider a change in policy* $\tau^t = \tau^{t*} + \varepsilon \tau^t_\varepsilon, G^t = G^{t*} + \varepsilon G^t_\varepsilon$, *indexed by* ε. *Then,*

[50] As in the main text, this condition corresponds to the the bias and homophily effects (now profit inclusive) being zero for all output and transfer shocks.

under assumptions 7 and 8, the following first-order condition holds:

$$
\begin{aligned}
0 = & \underbrace{\left(\widetilde{\lambda}^T \widehat{\mu} WTP^1 - (\gamma \vec{1}^T + \widetilde{\lambda}^T \check{\Delta} \check{R}^1)\right) G_\varepsilon^1}_{\textit{Opportunistic government spending}} + \underbrace{\frac{\left(\widetilde{\lambda}^T \widehat{\mu}(I - \widehat{\phi}) WTP^2 - \gamma \vec{1}^T\right) G_\varepsilon^2}{1 + r^1}}_{\textit{Short-termist government spending}} \\
& \underbrace{- (\widetilde{\lambda} - \gamma \vec{1})^T \widehat{\mu} \left(\tau_\varepsilon^1 + \frac{\tau_\varepsilon^2}{1 + r^1}\right)}_{\textit{Pure redistribution}} \underbrace{+ \widetilde{\lambda}^T \frac{\widehat{\phi} \widehat{\mu} \tau_\varepsilon^2}{1 + r^1}}_{\textit{Relaxation of borrowing constraints}} \\
& \underbrace{- \widetilde{\lambda}^T \check{\Delta} \check{R}^1 \left(I - \check{C}_{y^1}^1 \check{R}^1\right)^{-1} \check{C}_{y^1}^1 \left(\check{R}^1 G_\varepsilon^1 - \widehat{\mu} \tau_\varepsilon^1 - \frac{1_{\phi_n = 0} \widehat{\mu} \tau_\varepsilon^2}{1 + r^1}\right)}_{\textit{Keynesian stimulus (alleviation of involuntary unemployment)}}
\end{aligned}
\tag{OA98}
$$

where γ is the marginal value of public funds, $\check{R}^1 = \begin{bmatrix} l_{L^1}^1 \widehat{L}^1 \\ \widehat{\Pi}^1 \end{bmatrix} \left(I - \widehat{X}^1\right)^{-1}$, $\check{C}_{y^1}^1 = \begin{bmatrix} C_{y^1}^1 & C_{y^1}^1 \end{bmatrix}$, and $\check{\Delta}$ is the $N \times 2N$ matrix with entries $\check{\Delta}_{n,n} = \Delta_n$, $\check{\Delta}_{n,N+n} = -1$, and zeros elsewhere.

Proof. This follows from reinterpreting profit income as labor supply with wedge -1 and then following the proof of Proposition 13. □

Intuitively, the planner targets "profit-wedges" just like labor supply wedges. These reduce the social cost of government spending and motivate Keynesian stimulus.

Finally, a similar network-irrelevance result holds as in the case without profits.

Proposition 16. *Impose Assumptions 7 and 8. Now, suppose that all households rationed to on the margin at the optimum have no marginal labor disutility, i.e. if $(R^1 C_{y^1}^1)_{n,-} \neq \vec{0}$ then $\Delta_n = 0$. Then Equation OA98 holds with respect to variations in first-period transfers if and only if, for all $n \in N$,*

$$
\gamma = \frac{\widetilde{\lambda}_n}{1 - m_n} \tag{OA99}
$$

Alternatively, suppose that the social gains from first-period government expenditure are equal to some $\tilde{v}$ across goods and constraints bounding expenditures above zero do not bind. Then Equation OA86 holds with respect to first-period expenditure variations if and only if, $\forall i \in I$,

$$
\gamma = \tilde{v} + \frac{1}{1 - \widetilde{m}_i} \left(-\widetilde{\lambda \Delta}_i\right) \tag{OA100}
$$

where $\tilde{m}_i \equiv \left(m^T R^1\right)_i$ is the rationing-weighted average MPC in the production of good i and $\widetilde{\lambda \Delta}_i \equiv \left(\widetilde{\lambda}^T \check{\Delta} \check{R}^1\right)_i$ is the rationing-and-welfare-weighted average rationing wedge in the production of good i, for R^1 is as in Proposition 14 and $\check{R}^1$ and $\check{\Delta}$ are as in Proposition 15.

Proof. This follows from reinterpreting profit income as labor supply with wedge -1, following Online Appendix A.5, and using $\Delta_n = 0$ for marginal labor-suppliers in the transfer case. □

C. Additional figures

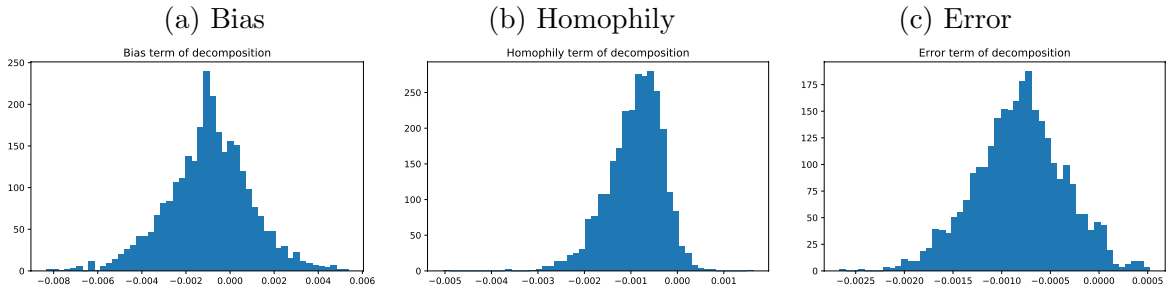

Fig. OA1. Histograms of the bias term (left) and homophily term (middle) and overall error terms (right) from the decomposition in Proposition 2. For all subfigures, the distribution reflects a unit demand shock to each of the 2805 sector-region pairs, with baseline y^* given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

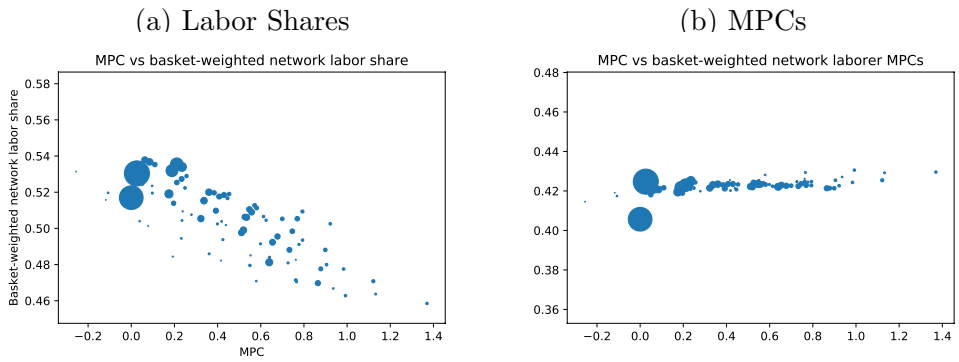

Fig. OA2. The left panel shows the scatter plot of worker MPCs against the basket-weighted labor share of the sectors on which they consume. The right panel shows a scatter plot of worker MPCs against the basket-weighted MPCs of the labor employed in the sectors producing the goods they ultimately consume.

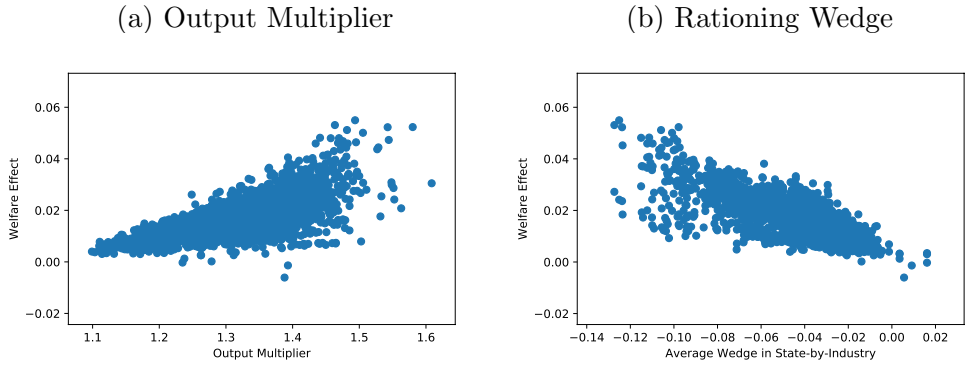

Fig. OA3. Left panel: The x-axis gives the output multiplier for a dollar of government spending targeting each of the 2805 state-industry pairs. The y-axis gives the estimated welfare effect of a dollar of government spending targeting each of the 2805 state-industry pairs using rationing wedges from the Great Recession. Right panel: The x-axis gives the population-weighted Great Recession rationing wedge of employees in each of the 2805 state-industry pairs. The y-axis gives the estimated welfare effect of a dollar of government spending targeting each of the 2805 state-industry pairs using rationing wedges from the Great Recession.